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# **Human Capital, Multiple Income Risk and Social Insurance**

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# Human Capital, Multiple Income Risk and Social Insurance<sup>\*</sup>

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## Abstract

We set up an OLG-model, where households both choose human capital investment and decide on investing their endogenous savings in a portfolio of riskless and risky assets, exposing them to (aggregate) wage and capital risks due to technological shocks. We derive the optimal public policy mix of taxation and education policy. We show that risks can be efficiently diversified between private and public consumption. This results hinges on that the government can apply a wide set of instruments, including differentiated wage and capital taxation. We also show that for sufficient risk aversion the (Northern) European way of relying on progressive wage taxation and granting education subsidies is an optimal response to wage and capital risks.

**JEL-Classification:** H21, I28, J24

**Keywords:** Optimal Income Taxation, Multiple Income Risks, Human Capital Investment, Portfolio Choice

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# 1 Introduction

In 1789, Benjamin Franklin stated in a letter to Jean-Baptist Leroy that “*in this world nothing can be said to be certain, except death and taxes.*” Households are in fact (and still) exposed to multiple risks in their lives – and among these, apart from the risk of falling seriously ill, wage and capital income risks probably are the most important risk factors for well-being: capital income is vulnerable to world-wide shocks, as there has been, e.g., a slow-down in stock markets and the real economy after the terror attack on 9/11 2001 as well as recently due to the US-subprime disaster. Wage income exhibits also large fluctuations, caused, among others, by globalization and skill-biased technological change.<sup>1</sup>

Another feature affecting well-being is education (or human capital). Whilst the importance of education is emphasized by many branches in the economic literature – very prominent is the one on human capital and growth (e.g., Bils and Klenow, 2000), – it has ambiguous and interdependent effects on income risk: studies and stylized facts show that human capital on the one hand acts as insurance against unemployment (Chapman, 1993, OECD, 2007), while on the other hand it amplifies other income risks (Mincer, 1974, Wildasin, 2000, Carneiro, 2003).

Unfortunately, human capital and wage risks are personalized, being non-tradable in markets. Consequently, households cannot diversify their exposure to risk efficiently. Moreover, even the idiosyncratic part of wage risks can rarely be insured against in private markets due to moral hazard, adverse selection and legal limitations (Sinn, 1996). Thus, what can be done in order to make life safer?

Almost 200 years after Franklin, it turned out in economic literature that the certainty of taxes can also have a welfare improving effect, because taxation and its revenue can provide risk insurance by decreasing the variance in income and consumption (i.e., Eaton and Rosen, 1980a,b). However, the previous literature – to the best of our knowledge – restricts to only one aspect of risk per model and to a limited set of governmental instruments.<sup>2</sup> Thus, it neglects combined effects of multiple income risk, faced by households and the fact that the government can

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<sup>1</sup>See Atkinson (2008) for a recent overview, showing several ups and downs in skilled and unskilled wages over the last century.

<sup>2</sup>An overview on related literature is provided in the next section.

use a wide set of instruments for public policy, including progressive taxation and educational policy.

Accordingly extending the scope of such optimal taxation models, the questions, emerging and being tackled in this paper, are: (i) What is the optimal tax structure in order to cope with multiple income risk in human and real capital, and in which way is public policy challenged by multiple risk? (ii) Which effects on the optimal trade-off between efficiency and insurance will emerge? (iii) Are direct or indirect instruments better to counter inefficiencies in human capital formation caused by taxation, hence, is it better to use tuition fees (or education subsidies) or capital taxation?

This paper shows that the government can provide efficient diversification of both (aggregate) wage and capital risk between private and public consumption, increasing the set of socially available assets, if leisure demand is inelastic, and if the government has access to differentiated wage taxation, tuition fees/education subsidies and a capital tax, which is only levied on the excess return in the risky asset.

Taxation of skilled wages and excess returns in real capital shifts risk into public consumption and decreases the variance in private consumption. The necessary tax rates increase in private relative to public risk aversion. Whilst the exact tax structure depends on this risk aversion in private and public consumption, riskless interest income should not be taxed in any case. If risk aversion is sufficiently high, we will end up with progressive wage taxation and education subsidies, backing most (Northern) European tax and education systems. Endogenizing leisure demand will complicate the analysis very much, and explicit solutions can hardly be derived. Instead, we provide some intuitive conjectures, based on results in simplified models.

For providing this missing link in normative theory of taxation under uncertainty, we set up an OLG-model, where households live for two periods. In their first period of life, they decide on educational investment, on overall savings and on portfolio choice between a risky and a riskless asset. Furthermore, they supply unskilled labor. In their second period, households are faced by risk in both return to real capital and wage income, they receive as skilled workers. The risks are due to stochastic technological shocks, which can increase the productivity of real

capital, but can also cause depreciations in the capital stock. Labor productivity is affected indirectly by the change in capital productivity and directly by the fact that technological progress can either be complementary to households' skills or depreciate their stock of human capital, in case households cannot handle the new technology.

The government is supposed to provide funding for the educational sector and to supply a public consumption good. For financing its expenditure, it can apply both differentiated taxation of wage income and capital taxes, which are tailored to specific parts of return – i.e., the excess return, which can be seen as the market price of aggregate capital risk. Moreover, it can rely on educational policy, as there are tuition fees or education subsidies.

The paper is organized as follows: In section 2 we provide a discussion on income risks, on the linkage to human capital and on the possibilities of the government to insure these risks. Section 3 then presents the model and is followed by the description of household behavior. In section 5 we derive the optimal public policy in case of inelastic leisure demand, whereas extensions and omissions are discussed in section 6. Section 7 closes with some conclusions.

## **2 Income Risks and Social Insurance by Taxation**

Sources of capital risk are manifold: it can be caused by business fluctuations and therefore by an uncertain profitability of the firm, by technological change, which can increase capital productivity, but also may cause extraordinary depreciations in the capital stock. For financial assets, risk can also be due to speculative shocks within financial capital markets. Sometimes, shocks causing world-wide turmoil in stock markets can spread out and slow down the real economy, e.g., after the terror attack on 9/11 2001 and the recent disaster in the US-subprime credit market. However, aggregate capital risk can still be traded in these capital markets, and each household can adjust its exposure to risk. Moreover, unsystematic capital risk can be entirely diversified.

The sources of wage risk are even more various: wage risk can also be caused by business cycles and technological progress may both increase or decrease the productivity of (skilled) labor, as well as depreciate the stock of skills. There is

incidence for an increasing wage gap in skilled and unskilled labor for the last 25 years, driven by globalization and skill-biased technological change (Krugman, 1995, Katz and Autor, 1999). This trend in wage changes is predicted to continue even more in the future, due to the growth rate of skilled labor supply lacking behind the demand for skilled workers (Jacobs, 2004) – implying that wages can be even less forecasted than by now. Atkinson (2008) provides a recent and critical overview on these topics and shows that there is a strong upward trend in skilled wages, whereas unskilled wages remain constant or decrease slightly. Moreover, he shows that there have been several increases and decreases in both skilled and unskilled wages over the last century.

However, there is an important difference to capital risk: wage risk can neither be (fully) insured nor traded in private markets.<sup>3</sup> Insurance fails due to, e.g., moral hazard or adverse selection and due to the fact that most people are (legally) too young for signing binding contracts, when they decide on investing in skilled labor. Trading is impossible, because manpower cannot serve as collateral – at least as long as slavery is precluded. This risk can therefore also hardly be diversified.

This implies that households must additionally bear idiosyncratic (success) risk in human capital formation and firm-specific risk at their employers. The latter implies that an employee additionally has to bear the idiosyncratic risk that its employer either goes bankrupt or cuts wages in order to avoid mass layoffs. These ex-post wage cuts have been very popular, e.g., in Germany for the last decade.<sup>4</sup> Taken together, there is large variety in wage risk, ranging from unemployment risk to productivity risk – and even wages, which are fixed ex ante, can be risky due to the mentioned wage renegotiations.

Higher education is often recommended as substitute for wage insurance. However, it cuts both ways: On the one hand human capital is in fact an insurance against the risk of getting unemployed. Empirical data shows that among unskilled workers unemployment is – on OECD- respectively on EU19-average – twice as high as among workers with a degree in upper-secondary education and

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<sup>3</sup>See e.g., Eaton and Rosen (1980b), pp. 707. A comprehensive overview on these arguments is contained in Sinn (1996). Even public unemployment insurance does not offer full coverage.

<sup>4</sup>Wage cuts have been mostly implemented by reducing gratifications like Christmas or vacation bonus. Wage reduction options are meanwhile a common tool in contracts between trade unions and employers in Germany.

even threefold higher than among skilled workers, having finished tertiary education (OECD, 2007, Indicator A8). On the other hand human capital investment is accompanied by the risk to fail in graduation<sup>5</sup> as well as it promotes, e.g., occupational risk and the risk of having highly specialized knowledge, which can only be used in few sectors – consequently, exposing its owner to sector-specific risks (Wildasin, 2000). Increasing (changing) wage differentials also render returns to education risky, fitting to the prediction by Mincer (1974). In fact, Carneiro et al (2003) back the view that graduate wages and returns cannot be predicted at the time of making the investment into human capital.<sup>6</sup>

Levhari and Weiss (1974) are first to analyze the effect of a variety of these wage risks onto human capital investment, while Williams (1978) extends the analysis onto multiple wage and capital income risk. Both papers show that these risks have a major impact on household behavior. Furthermore, the latter points out that investment in human capital and portfolio choice in real capital assets are strongly linked, if returns to both investments are risky. These papers do, however, neither deal with public policy nor with insurance possibilities (except for under-/overinvestment as self-insurance).

Public policy is such a possibility for improving the allocation of risk and for providing some insurance even in those cases, where the private sector will not supply insurance against income risk (see Sinn, 1996): By reducing the variance in ex-post incomes via taxation and by redistributing tax revenue as deterministic transfers in case of idiosyncratic risk respectively by diversifying aggregate risk between private and public consumption, taxation can insure these risks. In the former case, the government can eliminate risk by pooling, thus it bears the risk at no costs – as long as we abstain from induced distortions.<sup>7</sup>

The latter case is somewhat more complicated. It appears somehow odd that the government should be able to deal better with aggregate risk than the private market, as long as one restricts to public projects, which could (in principle) also be realized by the private market. There has been a lively debate on that issue,

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<sup>5</sup>Drop-out rates are substantial, being for tertiary education around 30% on OECD-average. See OECD (2007), Indicator A3 and Table A3.6.

<sup>6</sup>Another short overview on the interdependency of human capital and various kinds of risk is to be found in Anderberg and Andersson (2003).

<sup>7</sup>See Varian (1980) for a detailed discussion.

and meanwhile it seems widely accepted that for such projects it is reasonable to assume that private and social valuation and discount factors should be equal.<sup>8</sup> Nevertheless, the government can improve the allocation of aggregate risk by supplying a public good, which is not provided by the market and therefore implement a public project, which is not contained in the private sector.<sup>9</sup> This holds true even if households are entirely diversified in all *private* assets, because the public good augments the number of *social* assets and therefore allows to spread risk onto more securities.

Optimal risk diversification then implies that aggregate risk is balanced on private and public consumption. In a first-best optimum, public insurance guarantees that the ex-post realized marginal utilities of private and public consumption are identical in each state of the world, what can be ensured by using state-dependent lump-sum taxes.<sup>10</sup>

If state-dependent lump-sum taxes are not available, a trade-off is emerging between risk diversification and potential distortions. There are several studies, characterizing second-best optima for different kinds of risk and for a limited set of public policies: Eaton and Rosen (1980a,b) as well as Hamilton (1987) point out in case of idiosyncratic wage risk that proportional income taxation and lump-sum transfers show the mentioned welfare-enhancing insurance effects.<sup>11</sup> In a second-best optimum, these insurance effects are balanced against induced distortions in labor supply and human capital formation. In Hamilton (1987), moreover, capital taxation can also serve as indirect instrument to correct for distortions in human capital investment. Based on the Hamilton-model, da Costa and Maestri (2007) follow a ‘new dynamic public finance’-approach and apply a wide set of non-linear instruments. Focusing on implicit tax wedges, they show wage and capital taxation to be desirable, whereas education investment should remain undistorted. However, an informative optimal tax structure is hard to derive from

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<sup>8</sup>See, e.g., Arrow and Lind (1970) vs. Hirshleifer (1966), Sandmo (1972) or Bailey and Jensen (1972), whereby the latter denote the assumption of risk neutrality in this case as ‘nirvana approach,’ because of comparing apples and oranges due to different institutional settings.

<sup>9</sup>See Kaplow (1994), p. 795 and Myles (1995), p. 210f.

<sup>10</sup>See, e.g., Christiansen (1995) or Gollier (2001), who relates the sensitivity of consumption to absolute risk tolerance (p. 313f and Proposition 80).

<sup>11</sup>Varian (1980) shows similar results in a model with risky return to capital investment and generalizes the result to non-linear income taxation.



their tax wedges and implicit tax rates.

In case of business risk, Kanbur (1980) models the occupational choice decision between working as an employee for a deterministic wage, or becoming entrepreneur and being faced by risk. In the second-best optimum, partial social insurance by differentiated taxation of both types of workforce is balanced against distortions in occupational choice.

For (idiosyncratic) risky human capital formation, García-Peñalosa and Wälde (2000) examine a broader range of instruments. Basically, they show that a graduate tax, accompanied by some direct education subsidies, is optimal in order to insure individuals. However, they restrict to a binary risk model, where students are either successful in investing or not and model the graduate tax as a lump-sum payment of all graduated households.

A more detailed linkage between wage risk, distortionary taxation, and education policy provide Anderberg and Andersson (2003), examining the effect of several types of wage risk onto tax revenue and welfare. They state that it is optimal to overprovide education, if human capital has an insurance function. However, in their model the government can control all human capital investment by mandatory education, and there is no private investment decision.

Turning to capital risk, a methodologically corresponding framework to Kanbur (1980) can be applied in case of portfolio choice. As shown in Christiansen (1993), there is an optimal trade-off between distorting investment in risky and riskless assets and the diversification of aggregate risk between private and public consumption by implementing differentiated asset-specific tax rates.

Put together, it is neglected to the best of our knowledge in this literature that households face simultaneously capital and wage risks for different reasons. The only study focusing on this issue and including human capital investment seems to be the work by Williams (1978). The optimal public policy in such a case has never been examined. Additionally, the cited studies restrict to a limited set of public instruments. This must have effects both on the ability to diversify risk and on the efficiency costs.

Modeling the effect of multiple risk and enlarging public instruments for differentiated wage taxation and tuition fees as well as capital taxes focusing on the excess return is the challenge to be tackled in the section to come.

### 3 The Model

We assume a small open economy with overlapping generations. In each generation there is a continuum of homogenous households. Each household lives for two periods, supplies unskilled labor in its first period of life and invests in real and human capital. Real capital is internationally perfectly mobile, whereas labor force is entirely immobile.

As each individual lives for two periods, overall population in period  $t$  is equal to  $N_t^{t-1} + N_t^t$ . Superscript  $t - 1$  indicates the old generation, born in period  $t - 1$ , whereas superscript  $t$  represents the actual young generation in period  $t$ . Furthermore, we assume constant and exogenous population growth at rate  $\eta$ , which is equal to the riskless interest rate  $r$ .  $\eta = r$  guarantees the ‘golden rule’ of real capital accumulation and avoids – without loss of generality – any intertemporal fiscal externality stemming from dynamical inefficiency (see, e.g, Atkinson and Sandmo, 1980, or Sandmo, 1985, p. 292).

**Production Sector** The domestic industry produces a homogenous consumption good  $y$ , whose price is normalized to unity. Production can take place in two sectors: sector 0, exhibiting both deterministic output and costs, and sector 1, which uses a risky production technology.<sup>12</sup>

In the deterministic sector 0, the representative firm issues riskless bonds  $I^0$ , which pay out return  $r$  in order to attract real capital  $K^0$ , and the firm demands unskilled labor  $L^0$ . It uses a constant-returns-to-scale production function,  $y^0 = F^0(K^0, L^0)$ . The riskless interest rate is then determined by perfect capital mobility and the production function as  $F_K^0 = r$ , where  $F_K^0$  is the marginal productivity of real capital in sector 0. Moreover, international capital flows enforce a wage rate for unskilled labor of  $F_L^0 = W^0$ .

The risky sector 1 utilizes always the latest production technology, which depends on a stochastic technology parameter  $\theta$ . In each period, there is a capital-augmenting technological shock, which can on the one hand increase the productivity of capital, but on the other hand also affects depreciations  $\delta$  either positively

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<sup>12</sup>The basic set-up equals Stiglitz (1972), and extends his model for both skilled and unskilled labor as well as endogenous human capital formation.

or negatively. Moreover, this technology requires skilled labor  $H$  to be used. The production function then takes the form  $y^1 = F^1(K^1, H, \theta)$ .

The representative firm issues stocks  $I^1$ , which deliver a stochastic return  $\tilde{x}$  in order to attract venture capital for production. Employment of capital follows from marginal productivity equal to capital costs. This can be rearranged to  $F_K^1(K^1, H, \theta) - \delta(\theta) = \tilde{x}$ . In the good states of the world, capital productivity is increased by the technological shock and depreciations are low, resulting in a high return to venture capital. In the bad states of the world, however, capital productivity is unaffected or even lowered by the shock, and it turns out that the capital stock has fully depreciated at the end of the production process. If this happens, the return to capital turns out to be negative or capital is even lost entirely. Taken together, the return to venture capital has in principle support  $\tilde{x} \in [-1; \infty]$ .

Accordingly, we obtain the optimal demand for human capital from  $F_H^1(K^1, H, \theta) = \tilde{W}^1$ . The marginal productivity of human capital depends twofold on the technological shock: First, there is an indirect effect via the productivity of capital. If the utilization of real capital changes, this should also affect the productivity of and the demand for human capital. Second, there is also a direct effect, which is independent of the productivity change in real capital. The productivity of human capital is directly affected by the capability to utilize the new technology. It may turn out that the qualifications of skilled workers are not sufficient in order to handle the new technology properly, or it might happen that the new technology is easier to cope with given a certain type of qualification. Thus, even if the shock increases (decreases) real capital productivity worldwide, it may occur in some countries that human capital productivity decreases (increases). This direct effect is a country-specific shock and is driven by differentiated education systems, where different skills might be acquired across countries. If marginal productivity of skilled workers becomes too low, however, they can supply their labor force in the riskless sector. In the riskless sector, human capital is useless, and the skilled just imitate the unskilled. Taken together, the wage rate of skilled labor has support  $\tilde{W}^1 \in [\frac{W^0}{g(E)}; \infty]$ .

**Households** The risk averse households are provided with one unit of time per period. In their first period of life, they decide to spend time  $e$  at university in

order to accumulate human capital. Time  $1 - e_{t-1}$  is supplied at wage rate  $W^0$  as unskilled labor. Hence, pre-tax income in that period is  $W^0 \cdot (1 - e_{t-1})$ . First-period income is split on consumption  $c_{t-1}$ , and savings  $s_{t-1}$ . Hereby, savings can be allocated in two assets: the amount  $A_{t-1}^0$  is invested in riskless bonds, which deliver a return  $r$  before capital taxation; the amount  $A_{t-1}^1$  is invested in a risky asset, which supplies the risky production sector with real capital. It pays out a stochastic pre-tax return  $\tilde{x}$ , being due to aggregate risk. Overall savings can be written as  $s_{t-1} = A_{t-1}^0 + A_{t-1}^1$ .<sup>13</sup>

In their second period of life, labor supply of households is inelastic and they supply one unit of time. If they are employed in the risky sector, their effective labor supply in units of skilled labor depends on the amount of human capital acquired. Human capital is accumulated according to a concave production function  $g(e)$  and increases in the time spent at university, that is  $g'(e) > 0$ ,  $g''(e) < 0$  and  $\tilde{g}(0) = 1$ . Thus, effective human capital supply is  $g(e)$ , labor market equilibrium implies  $H_t = g(e_{t-1})$ , and pre-tax labor income in the second period equals  $\tilde{W}^1 \cdot g(e)$ . The latter is risky in aggregate, due to a stochastic wage rate  $\tilde{W}^1$ . The lower bound of labor income is the unskilled wage income  $W^0$ , because if the marginal productivity in human capital and skilled labor income becomes too low,  $g(e) \cdot \tilde{W}^1 < W^0$ , the skilled households decide to work in the riskless sector 0. Here they cannot utilize their human capital and supply one unit of labor at the unskilled wage rate  $W^0$ .

Following the mainstream of the literature, we will assume that wage risks in human capital can neither be insured against nor can be traded (see Sinn, 1996). In any case, consumption when old,  $C_t$ , has to be financed from two risky earnings bases, namely stochastic labor income and risky capital income.

**Government** The government on the one hand provides a pure public consumption good  $P_t$ . On the other hand, the government also has to provide a public higher education system, which causes real resource costs  $\bar{B}$  per student. This expenditure is assumed to be fixed per student and independent from time investment  $e$ . The government charges, however, a price  $p_B$  per semester and can exclude stu-

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<sup>13</sup>Using  $m$  as a country index, world capital market equilibrium then implies  $\sum_m A_m^0 = \sum_m I_m^0$  and  $\sum_m A_m^1 = \sum_m I_m^1$  in each period of time.

dents, who are not willing to pay  $p_B$  per unit of time spent at university,  $e$ . This price for education can be seen as tuition fees per semester, if  $p_B > 0$ , or it will turn into education subsidies, if  $p_B < 0$ . The overall net public expenditure for education in period  $t$  is then given by

$$B_t^{net} = N_t^t \cdot (\bar{B} - p_B \cdot e_t). \quad (1)$$

Taken together, overall public expenditure in period  $t$  is

$$R_t = P_t + B_t^{net} = P_t + N_t^t \cdot (\bar{B} - p_B \cdot e_t). \quad (2)$$

In order to finance its expenditure, the government can use a set of labor and capital income taxes. For labor taxation, we apply a Norwegian-type two-bracket tax schedule as in Nielsen and Sørensen (1997): All labor income until a threshold  $X = W^0$  is liable to the labor tax rate  $t_1^L$ . The part of labor income, exceeding this threshold, consequently the skill premium  $\tilde{W}^1 \cdot g(e) - W^0$ , is liable to the labor tax rate  $t_2^L$ . Therefore, unskilled workers are only faced by the tax rate  $t_1^L$ , whereas the marginal tax rate of the skilled ones is equal to the surtax rate  $t_2^L$ .

Capital taxation is also differentiated: Riskless capital income in *both* assets is taxed at rate  $t_0^K$ . The excess return in the risky asset,  $\tilde{x} - r$ , thus the price received for incurring risk, is taxed instead at rate  $t_1^K$ . In the latter tax base, full loss offset is guaranteed. This implies a refund of  $t_1^K \cdot (\tilde{x} - r)$  per unit of risky capital investment,  $A^1$ , if  $\tilde{x} - r$  turns ex-post out to be negative. The modeling of the capital tax corresponds to the Norwegian shareholder income tax and allows to tax capital risk directly (see Sørensen, 2005, and Schindler, 2008).

**Risk in the Economy and Timing Structure** There are two different income risks in the economy, which depend both on the technology shock. First, this shock can be seen as capital-augmenting technological progress. However, it is ex-ante uncertain, whether production is really enhanced and what the effects on depreciation costs are. We assume that this shock strikes all firms in the risky sector in all countries at the same time and in the same manner. Hence, the shock cannot be insured and it translates into aggregate income risk for stock holders.

Second, the technological shock affects human capital in the risky sector

twofold: (i) There is an indirect effect via the productivity of venture capital. It seems reasonable to assume that the productivity of skilled labor is *ceteris paribus* increased (decreased), if the productivity of real capital increases (decreases). (ii) There is also a direct impact of the technological shock. We assume that the capability of skilled labor to utilize the new technology depends on the skills acquired at university and differs across the countries. The reasoning behind this is the implicit assumption that there are international differences in the educational systems. Accordingly, this corresponds to an asymmetric shock. In some countries human capital productivity may be enhanced, whereas, in extremum, in some other, few, countries, the skilled workers cannot use the new technology at all. In the latter case, there will be no production in the risky sector and all skilled workers will supply one unit of unskilled labor in the deterministic sector. As labor force is internationally immobile, human capital risk still cannot be insured against (internationally). Hence, the effects of the technological shock translate into aggregate labor income risk for skilled workers as well. From the government point of view, both the labor income tax base and the capital income tax base are partly risky, and, thus, overall tax revenue is stochastic, too.

The timing structure and the realization of risk is as follows: First, the benevolent government sets welfare-maximizing tax rates and tuition fees. Second, the young generation decides for its human capital investments, optimal savings and portfolio allocation. Next, the impact of the technological shock  $\theta$  on venture and human capital realizes, real capital is allocated worldwide, and the skilled workers decide to work either in the risky sector or in the deterministic one. Then, production takes place, and the real value of depreciation in venture capital,  $\delta(\theta)$  realizes. Finally, all incomes and taxes are paid, and private as well as public consumption take place.

## 4 Household Choice

An individual, born in period  $t - 1$ , maximizes its von-Neumann-Morgenstern expected utility function

$$Z = E[U(c_{t-1}, c_t)] + E[V(\tilde{P}_t)] \quad (3)$$

by choosing its optimal educational investment  $e_{t-1}$ , its consumption  $c_{t-1}$  and its investments in the riskless and the risky financial asset,  $A_{t-1}^S$  and  $A_{t-1}^R$ , respectively.

We assume the utility function to be additively separable in private and public consumption. Moreover, the individual does not anticipate any effects of its behavior on the level or the riskiness of the public good, because each household is arbitrarily small.

The budget constraint of the household under consideration is in period  $t - 1$  given by

$$(1 - t_1^L) \cdot W_{t-1}^0 \cdot (1 - e_{t-1}) = c_{t-1} + p_B \cdot e_{t-1} + A_{t-1}^S + A_{t-1}^R, \quad (4)$$

and human capital formation and savings translate into second-period-of-life consumption<sup>14</sup>

$$\begin{aligned} \tilde{c}_t = & (1 - t_2^L) \cdot [\tilde{W}^1 \cdot g(e_{t-1}) - W^0] - t_1^L \cdot W^0 + (1 - t_1^K)(\tilde{x} - r) \cdot A_{t-1}^R \\ & + [1 + r(1 - t_0^K)] \cdot [A_{t-1}^S + A_{t-1}^R]. \end{aligned} \quad (5)$$

Consolidating these expressions leads to the intertemporal budget constraint

$$\begin{aligned} \tilde{c}_t = & (1 - t_2^L) \cdot [\tilde{W}^1 \cdot g(e_{t-1}) - W^0] - t_1^L \cdot W^0 + (1 - t_1^K)(\tilde{x} - r) \cdot A_{t-1}^R \\ & + [1 + r(1 - t_0^K)] \cdot [(1 - t_1^L) \cdot W_{t-1}^0 \cdot (1 - e_{t-1}) - c_{t-1} - p_B \cdot e_{t-1}], \end{aligned} \quad (6)$$

whereby  $(1 - t_1^L) \cdot W_{t-1}^0 \cdot (1 - e_{t-1}) - c_{t-1} - p_B \cdot e_{t-1} = A_{t-1}^S + A_{t-1}^R = s_{t-1}$  are overall savings.

Thus, the household solves

$$\max_{c_{t-1}, A_{t-1}^R, e_{t-1}} E[U(c_{t-1}, \tilde{c}_t)] + E[V(P_t)] \quad \text{s.t.} \quad (6). \quad (7)$$

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<sup>14</sup>All variables indicated with a tilde depend on the realization of  $\theta$  and are stochastic.

First order conditions are

$$E[U_{c_{t-1}}] - p \cdot E[U_{c_t}] = 0 \quad (8)$$

$$(1 - t_1^K) \cdot E[U_{c_t} \cdot (\tilde{x} - r)] = 0 \quad (9)$$

$$E[U_{c_t} \cdot \{(1 - t_2^L) \cdot \tilde{W}_t^1 \cdot g'(e_{t-1}) - p \cdot ((1 - t_1^L) \cdot W^0 + p_B)\}] = 0, \quad (10)$$

where  $p = 1 + r(1 - t_0^K)$ .

From (8) we infer the usual condition that the marginal rate of time preferences,  $\rho = E[U_{c_{t-1}}]/E[U_{c_t}] - 1$ , must be equal to the riskless after-tax interest rate, accordingly  $\rho = r(1 - t_0^K)$ .

First order condition (9) implies that the risk tax  $t_1^K$  on the excess return in the risky financial asset only has a Musgrave-substitution effect,<sup>15</sup>

$$\frac{\partial A_{t-1}^R}{\partial t_1^K} = \frac{A_{t-1}^R}{1 - t_1^K}, \quad (11)$$

which reduces return, variance and all higher moments in the same way and which does not affect welfare from private consumption. Therefore it has neither effect on consumption  $c_{t-1}$  nor on educational investment  $e_{t-1}$ . Thus, we have  $\frac{\partial c_{t-1}}{\partial t_1^K} = \frac{\partial e_{t-1}}{\partial t_1^K} = 0$ . All of this can easily be understood by using the optimal investment function  $A_{t-1}^R(t_1^K) = \frac{A_{t-1}^R}{1 - t_1^K}$  in the household budget constraint (6).

Last, but not least, we draw from (10) that the effective risk-adjusted marginal return to human capital will be equalized to the after-tax marginal return in riskless real capital and

$$\frac{(1 - t_2^L) \cdot (1 - \pi_c(\tilde{W}^1)) \cdot \tilde{W}^1 \cdot g'(e_{t-1})}{(1 - t_1^L) \cdot W^0 + p_B} - 1 = r(1 - t_0^K), \quad (12)$$

whereby we have been using the certainty equivalent

$$W_{adc}^1 = \frac{E[U_{c_t} \cdot \tilde{W}^1]}{E[U_{c_t}]} = E[\tilde{W}^1] + \frac{\text{Cov}(U_{c_t}, \tilde{W}^1)}{E[U_{c_t}]} = \tilde{W}^1 \cdot (1 - \pi_c(\tilde{W}^1)), \quad (13)$$

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<sup>15</sup>This effect is well-known in the literature on risk taking and taxation. See, e.g., Mossin (1968), Sandmo (1969, 1977).



and  $\bar{W}^1 = E[\tilde{W}^1]$ , as well as  $\pi_c(\tilde{W}^1) = -\frac{\text{Cov}(U_{c_t}, \tilde{W}^1)}{E[U_{c_t}] \cdot \bar{W}^1}$ .  $\pi_c(\tilde{W}^1) \in (0, 1]$  is the normalized risk premium demanded in private consumption in order to bear the wage risk of an high-skilled worker. It acts like an implicit tax on (expected) skilled wage income.

From (13) and the first order condition (9) we can also infer an effect of the fact that human capital risk cannot be traded, whilst risk in real capital can be sold and bought via the risky asset. Equation (9) implies that the household is perfectly diversified in all real capital assets, because in the optimum the risk adjusted return of another marginal unit in the risky asset equals exactly the return in the riskless asset. By rearranging the optimality condition, we receive

$$E[\tilde{x} - r] = -\frac{\text{Cov}(U_{c_t}, \tilde{x})}{E[U_{c_t}]} = RP_c(\tilde{x}). \quad (14)$$

The certainty equivalent is given by the riskless market return. The household's absolute risk premium in real capital,  $RP_c(\tilde{x})$ , can therefore be inferred from market data,  $E[\tilde{x} - r]$ , and taxing the excess return  $\tilde{x} - r$  allows to tax the risk premium itself.

Transforming (13), the absolute risk premium in human capital is equal to

$$RP_c(\tilde{W}^1) = \bar{W}^1 \cdot \pi_c(\tilde{W}^1) = -\frac{\text{Cov}(U_{c_t}, \tilde{W}^1)}{E[U_{c_t}]} = \bar{W}^1 - W_{adc}^1, \quad (15)$$

but market data does not provide any information on the certainty equivalent  $W_{adc}^1$ .

The skill premium  $\tilde{W}^1 \cdot g(e) - W^0$  can be seen as a possible approximation for tax purposes, but it still mixes up the expected return to human capital and its risk premium. Thus, it seems not to be possible to tax the risk premium in wage income alone.<sup>16</sup> Moreover, it indicates that the household is not able to diversify the wage risk entirely.

Optimal household behavior determines the indirect utility function

$$\Omega(t_1^L, t_2^L, t_0^K, t_1^K, p_B) = E[U(c_{t-1}^*, c_t^*)] + E[V(\tilde{P}_t)], \quad (16)$$

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<sup>16</sup>Of course, it is possible to solve equation (12) for the risk premium  $RP_c(\tilde{W}^1)$ , but this will not deliver a suitable tax base.

and applying the Envelope-theorem leads to

$$\frac{\partial \Omega}{\partial t_1^L} = -W^0 \cdot [1 + p \cdot (1 - e_{t-1}^*)] \cdot E[U_{c_t}] \quad (17)$$

$$\begin{aligned} \frac{\partial \Omega}{\partial t_2^L} &= E[U_{c_t} \cdot (W^0 - \tilde{W}^1 \cdot g(e_{t-1}^*))] \\ &= -[(1 - \pi_c(\tilde{W}^1)) \cdot \tilde{W}^1 \cdot g(e_{t-1}^*) - W^0] \cdot E[U_{c_t}] \end{aligned} \quad (18)$$

$$\frac{\partial \Omega}{\partial t_0^K} = -r \cdot s_{t-1}^* \cdot E[U_{c_t}] \quad (19)$$

$$\frac{\partial \Omega}{\partial t_1^K} = -A_{t-1}^{R*} \cdot E[U_{c_t} \cdot (\tilde{x} - r)] = 0 \quad (20)$$

$$\frac{\partial \Omega}{\partial p_B} = -p \cdot e_{t-1}^* \cdot E[U_{c_t}], \quad (21)$$

where the second equality in equation (20) stems from the household first order condition (9) and confirms our arguments given above for the effects of  $t_1^K$  in comparative statics.

## 5 Optimal Public Policy

The government provides a pure public good,  $P_t = N_t^{t-1} \cdot G_t$ , and also has to provide a higher education system, publicly financed, which causes fixed costs  $\bar{B}$  per student. Whilst the level of the public good can vary, dependent on tax revenue, the education system must be fully funded in each state of nature.

Subtracting revenue from tuition fees, the overall (net) public expenditure for education in period  $t$  is given by

$$B_t^{net} = N_t^t \cdot (\bar{B} - p_B \cdot e_t^*). \quad (22)$$

Summed up, overall public net expenditure in period  $t$  is

$$\tilde{R}_t = N_t^{t-1} \cdot \tilde{G}_t + N_t^t \cdot (\bar{B} - p_B \cdot e_t^*), \quad (23)$$

whereby  $\tilde{G}_t$  are the units of the public good per member of the old generation.

In order to finance its expenditure, the government can use the set of wage and

capital income taxes stated in section 3. Labor income up to a threshold  $W^0$  is liable to the wage tax rate  $t_1^L$ . The part of labor income, exceeding this threshold, is liable to the wage tax rate  $t_2^L$ . Riskless capital income in both assets is taxed at rate  $t_0^K$ , whereas the excess return in the risky asset,  $\tilde{x} - r$ , is taxed at rate  $t_1^K$ . In the latter tax base, full loss offset is guaranteed.

All together, the government receives in each period  $t$  wage tax revenue  $N_t^t \cdot t_1^L \cdot W^0 \cdot (1 - e_t^*)$  from the young generation. The old generation pays wage taxes  $N_t^{t-1} \cdot t_1^L \cdot W^0$  at the standard rate and, additionally, has to pay  $N_t^{t-1} \cdot t_2^L \cdot [\tilde{W}^1 \cdot g(e_{t-1}^*) - W^0]$  under the surtax rate. The latter tax base is risky in aggregate, but as the income of a skilled worker cannot be lower than the wage paid in the riskless unskilled sector,  $W^0$ , this tax base cannot be negative, thus  $[\tilde{W}^1 \cdot g(e_{t-1}^*) - W^0] \geq 0$ .

The governmental budget restriction for period  $t$  is therefore given by

$$\begin{aligned} & N_t^{t-1} \cdot \{t_2^L \cdot [\tilde{W}^1 \cdot g(e_{t-1}^*) - W^0] + t_1^L \cdot W^0\} + N_t^t \cdot t_1^L \cdot W^0 \cdot (1 - e_t^*) + \\ & N_t^{t-1} \cdot \{t_1^K \cdot (\tilde{x} - r) \cdot A_{t-1}^R + t_0^K r \cdot [(1 - t_1^L) \cdot W^0 \cdot (1 - e_{t-1}^*) - p_B \cdot e_{t-1}^* - c_{t-1}^*]\} \\ & = \tilde{R}_t = N_t^{t-1} \cdot \tilde{G}_t + N_t^t \cdot (\bar{B} - p_B \cdot e_{t-1}^*). \end{aligned} \quad (24)$$

Rearranging and transforming into a per-capita constraint results in

$$\begin{aligned} & t_2^L \cdot [\tilde{W}^1 \cdot g(e_{t-1}^*) - W^0] + t_1^L \cdot W^0 + (1 + r) \cdot [t_1^L \cdot W^0 \cdot (1 - e_t^*) + p_B \cdot e_t^*] \\ & + t_1^K \cdot (\tilde{x} - r) \cdot A_{t-1}^{R*} + t_0^K r \cdot s_{t-1}^* - (1 + r) \cdot \bar{B} = \tilde{G}_t, \end{aligned} \quad (25)$$

where we used  $N_t^t / N_t^{t-1} = 1 + \eta = 1 + r$  and  $s_{t-1}^* = (1 - t_1^L) \cdot W^0 \cdot (1 - e_{t-1}^*) - p_B \cdot e_{t-1}^* - c_{t-1}^*$ . As the education system is always fully funded, the consumption of the public good  $\tilde{G}_t$  turns risky, as it is financed by risky tax revenue.

The government maximizes expected utility of a representative steady-state generation, born at  $t - 1$ .<sup>17</sup> Using the indirect utility function (16), the optimization problem can be stated as

$$\max_{t_1^L, t_2^L, t_0^K, t_1^K, p_B, \bar{B}} N_t^{t-1} \cdot \Omega(t_1^L, t_2^L, t_0^K, t_1^K, p_B, \bar{B}) + E[V(N_t^{t-1} \cdot \tilde{G}_t)], \quad (26)$$

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<sup>17</sup>This approach is compatible with a Pareto-improving tax reform as in Nielsen and Sørensen (1997), if we redefine expenditure  $\bar{B}$  and add debt payments necessary in order to keep the utility of the transition generation constant.

where  $\tilde{G}_t$  is subject to the budget restriction (25). Given the steady-state assumption, we are going to drop the superscripts for generations and time indices, whenever possible without causing confusion, in order to simplify the notation.

The first order conditions are

$$\begin{aligned} & -W^0 \cdot [1 + p \cdot (1 - e^*)] \cdot E[U_{c_t}] + \quad (27) \\ E \left\{ V_G \cdot \left[ W^0 [1 + p \cdot (1 - e^*)] + \tilde{\alpha} \cdot \frac{\partial e}{\partial t_1^L} + t_1^K \cdot (\tilde{x} - r) \cdot \frac{\partial A^R}{\partial t_1^L} - t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_1^L} \right] \right\} &= 0, \end{aligned}$$

$$\begin{aligned} & -[(1 - \pi_c(\tilde{W}^1)) \cdot \tilde{W}^1 \cdot g(e^*) - W^0] \cdot E[U_{c_t}] + \quad (28) \\ E \left\{ V_G \cdot \left[ \tilde{W}^1 g(e^*) - W^0 + \tilde{\alpha} \cdot \frac{\partial e}{\partial t_2^L} + t_1^K \cdot (\tilde{x} - r) \cdot \frac{\partial A^R}{\partial t_2^L} - t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_2^L} \right] \right\} &= 0, \end{aligned}$$

$$\begin{aligned} & -r \cdot s^* \cdot E[U_{c_t}] + \quad (29) \\ E \left\{ V_G \cdot \left[ r s + \tilde{\alpha} \cdot \frac{\partial e}{\partial t_0^K} + t_1^K \cdot (\tilde{x} - r) \cdot \frac{\partial A^R}{\partial t_0^K} - t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_0^K} \right] \right\} &= 0, \end{aligned}$$

$$\begin{aligned} & -p \cdot e^* \cdot E[U_{c_t}] + \quad (30) \\ E \left\{ V_G \cdot \left[ (\tilde{x} - r) A^{R*} + \tilde{\alpha} \cdot \frac{\partial e}{\partial t_1^K} + t_1^K \cdot (\tilde{x} - r) \cdot \frac{\partial A^R}{\partial t_1^K} - t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_1^K} \right] \right\} &= 0, \end{aligned}$$

$$\begin{aligned} & -p \cdot e^* \cdot E[U_{c_t}] + \quad (31) \\ E \left\{ V_G \cdot \left[ p e^* + \tilde{\alpha} \cdot \frac{\partial e}{\partial p_B} + t_1^K \cdot (\tilde{x} - r) \cdot \frac{\partial A^R}{\partial p_B} - t_0^K r \cdot \frac{\partial c_{-1}}{\partial p_B} \right] \right\} &= 0, \end{aligned}$$

whereby  $\tilde{\alpha} = t_2^L \cdot \tilde{W}^1 \cdot g'(e^*) - (1 + r) [t_1^L \cdot W^0 - p_B] - t_0^K r \cdot [(1 - t_1^L) \cdot W^0 + p_B]$  represents the (stochastic) net tax wedge on education, whilst  $c_{-1}$  indicates consumption in the first period of life and where we have already inserted the envelope-effects (17) – (21) for the derivatives of the indirect utility function.

As we have  $\frac{\partial e}{\partial t_1^K} = \frac{\partial c_{-1}}{\partial t_1^K} = 0$  and  $\frac{\partial A^R}{\partial t_1^K} = \frac{A^R}{1 - t_1^K}$  from (11) and comparative-statics, first order condition (30) simplifies to

$$E[V_G \cdot (\tilde{x} - r)] \cdot \frac{A_1}{1 - t_1^K} = 0 \quad \Leftrightarrow \quad E[V_G \cdot (\tilde{x} - r)] = 0. \quad (32)$$

Consequently, a marginal increase in the tax rate  $t_1^K$  will create additional tax revenue of  $\tilde{x} - r$ , however, in the optimum the risk adjusted value of this (additional) marginal tax revenue must be zero.

Next, we define analogous to  $W_{ad_c}^1$

$$W_{ad_G}^1 = \frac{E[V_G \cdot \tilde{W}^1]}{E[V_G]} = \frac{E[V_G] \cdot E[\tilde{W}^1]}{E[V_G]} + \frac{\text{Cov}(V_G, \tilde{W}^1)}{E[V_G]} = \bar{W}^1 \cdot (1 - \pi_G(\tilde{W}^1)). \quad (33)$$

$W_{ad_G}^1$  is the risk adjusted skilled wage, whereby the adjustment is now based on public consumption. It is equal to the expected skilled wage,  $E[\tilde{W}^1] = \bar{W}$ , minus the absolute risk premium measured in *public* consumption,  $RP_G(\tilde{W}^1) = -\frac{\text{Cov}(V_G, \tilde{g}(E))}{E[V_G]}$ .

Using equations (32) and (33) in the other first order conditions, we obtain

$$W^0 \cdot [1 + p \cdot (1 - e^*)] \cdot \frac{E[U_{c_t} - V_G]}{E[V_G]} = \beta \cdot \frac{\partial e}{\partial t_1^L} + t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_1^L}, \quad (34)$$

$$[W_{ad_c}^1 \cdot g(e^*) - W^0] \cdot \frac{E[U_{c_t}]}{E[V_G]} - [W_{ad_G}^1 \cdot g(e^*) - W^0] = \beta \cdot \frac{\partial e}{\partial t_2^L} + t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_2^L}, \quad (35)$$

$$r \cdot s^* \cdot \frac{E[U_{c_t} - V_G]}{E[V_G]} = \beta \cdot \frac{\partial e}{\partial t_0^K} + t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_0^K}, \quad (36)$$

$$p \cdot e^* \cdot \frac{E[U_{c_t} - V_G]}{E[V_G]} = \beta \cdot \frac{\partial e}{\partial p_B} + t_0^K r \cdot \frac{\partial c_{-1}}{\partial p_B}, \quad (37)$$

where  $\beta = t_2^L \cdot (1 - \pi_G(\tilde{W}^1)) \cdot \bar{W}^1 \cdot g'(e^*) - p \cdot (t_1^L \cdot W^0 - p_B) - t_0^K r$  is the risk-adjusted net revenue from taxing education.

Dividing equation (37) by  $p \cdot e^*$ , inserting the new expression in (36), and in (34) respectively, and rearranging those, reveals:

$$\begin{aligned} \left( p \cdot e^* \cdot \frac{\partial e}{\partial t_0^K} - r \cdot s^* \cdot \frac{\partial e}{\partial p_B} \right) \cdot \beta &= t_0^K r \cdot \left( p \cdot e^* \cdot \frac{\partial c_{-1}}{\partial t_0^K} - r \cdot s^* \cdot \frac{\partial c_{-1}}{\partial p_B} \right) \quad (38) \\ \left( p \cdot e^* \cdot \frac{\partial e}{\partial t_1^L} - W^0 [1 + p \cdot (1 - e^*)] \cdot \frac{\partial e}{\partial p_B} \right) \cdot \beta &= t_0^K r \cdot \left( p \cdot e^* \cdot \frac{\partial c_{-1}}{\partial t_1^L} - \right. \\ &\quad \left. W^0 [1 + p \cdot (1 - e^*)] \cdot \frac{\partial c_{-1}}{\partial p_B} \right) \quad (39) \end{aligned}$$

We can now state a first result:

**Proposition 1.** *It is not optimal to tax the riskless rate of return in financial assets.  $t_0^K = 0$  also implies that capital taxation is not used as indirect instrument to correct for labor-tax induced distortions in education demand. Moreover, education is not taxed on a net basis,  $\beta = 0$ .*

*Proof.* See Appendix 8.1. □

As it will turn out later, insurance and risk diversification is carried out by differentiated wage taxation and the risk tax on the excess return in risky assets.

Furthermore, educational investment can be controlled by using tuition fees / education subsidies. Hence, there is neither reason for distorting intertemporal consumption choice and educational investment nor any need for taxing (riskless) capital taxation and education. Accordingly, we have the marginal rate of intertemporal substitution equal to the riskless interest rate,  $\rho = r$ , and we have efficient (risk-adjusted) educational investment.

That the optimal tax system on the one hand actually does not create distortions, but on the other hand is still able to ensure the diversification of wage risk between private and public consumption in a very efficient manner, can be seen in Proposition 2:

**Proposition 2.** *Optimal public policy ensures (i) ex-ante efficiency in allocation,  $E[U_{c_t}] = E[V_G]$ , and (ii) an ex-post wage-risk sharing rule, which equates the (wage) risk premia in private and public consumption,  $\pi_c(\tilde{W}^1) = \pi_G(\tilde{W}^1)$ .*

*Proof.* Applying  $t_0^K = \beta = 0$  from Proposition 1, and  $e^* < 1$  in any of the equations (34), (36) or (37), results in

$$E[U_{c_t}] = E[V_G], \quad (40)$$

being exactly the definition of ex-ante efficiency.

Part (ii) can be proven by substituting  $t_0^K = \beta = 0$  in equation (35), where we obtain

$$[W_{ad_c}^1 \cdot g(e^*) - W^0] \cdot \frac{E[U_{c_t}]}{E[V_G]} - [W_{ad_G}^1 \cdot g(e^*) - W^0] = 0. \quad (41)$$

Applying  $E[U_{c_t}] = E[V_G]$  as well as the definition of  $W_{ad_a}^1 = \bar{W}^1 \cdot (1 - \pi_a(\tilde{W}^1))$ ,  $a = c, G$  and collecting terms then leads to

$$\pi_c(\tilde{W}^1) - \pi_G(\tilde{W}^1) = 0. \quad (42)$$

Thus, the wage risk premia in private and public consumption are equalized for an optimal public policy. From part (i) and the definitions of  $\pi_a(\tilde{W}^1)$ ,  $a = c, G$  follows as well

$$\text{Cov}(U_{c_t}, \tilde{W}^1) = \text{Cov}(V_G, \tilde{W}^1). \quad (43)$$

□

It is well-known that ex-ante efficiency itself should not be the aim of the government in case of risky economies and incomplete insurance markets (e.g., Eaton and Rosen, 1980a, Christiansen, 1993), because providing an insurance effect by taxation can compensate for tax-induced losses in efficiency. However, our broad set of instruments in combination with exogenous leisure choice simultaneously allows for both a very efficient diversification of risk, equalizing even the covariances itself, and, in expected terms, efficiency in allocation.

In fact, marginal utilities in private and public consumption are linked by a risk sharing rule, equating the (private and public) ‘prices’ of wage risk,  $\pi_c(\tilde{W}^1) = \pi_G(\tilde{W}^1)$ , and guaranteeing that marginal utilities fluctuate in a similar way, but not causing efficiency costs. Risk is shifted between private and public consumption by making use of the surtax rate. As will be stated in more detail later, the higher risk aversion in private consumption is, relative to the one in public consumption, the higher the tax rate  $t_2^L$  should be. The reason is that the more risk should be transferred into public consumption, because it is borne there at lower (utility) costs, then. Note, however, that, although this optimal policy is better than in standard optimal taxation models featuring wage risk (i.e., Eaton and Rosen, 1980b, Kanbur, 1980), the risk sharing rule cannot guarantee a first-best solution, because the risk is diversified in a linear manner, due to a constant surtax rate  $t_2$ .

Still it allows to transfer risk from the household to the government and attains therefore a twofold improvement. First, the household gets enabled to “trade” a part of its wage risk. Second, the government increases the number of (social) assets, onto which aggregate risk can be diversified, by providing a public good. The public good can be seen as an additional asset, which cannot be provided by the private markets.<sup>18</sup> This result does neither imply any assumption, whether the government can deal better with risk than private markets, nor does it require a statement, what the correct social discount rate should be.

Turning to insuring risk in real capital investment, we conclude that an equivalent risk diversification rule applies as implied by Proposition 2.

**Proposition 3.** *The optimal capital-risk sharing rule ensures that the normalized (capital) risk premia in private and public consumption are equalized in equilib-*

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<sup>18</sup>See Kaplow (1994), p. 795.

rium,  $\pi_c(\tilde{x}) = \pi_G(\tilde{x})$ .

*Proof.* We infer from equations (9) and (32) that

$$E[U_{c_t} \cdot (\tilde{x} - r)] = 0 = E[V_G \cdot (\tilde{x} - r)]. \quad (44)$$

Applying Steiner's Rule for covariances and  $E[U_{c_t}] = E[V_G]$  from (40), as well as rearranging, we come to

$$\pi_c(\tilde{x}) = -\frac{\text{Cov}(U_{c_t}, x)}{E[U_{c_t}] \cdot E[\tilde{x}]} = -\frac{\text{Cov}(V_G, x)}{E[V_G] \cdot E[\tilde{x}]} = \pi_G(\tilde{x}) \quad (45)$$

□

The risk in capital income is also diversified between private and public consumption in order to ensure that utility in private and public consumption are ex-post fluctuating in a desirable way – this time dependent on the realization of the risky return  $\tilde{x}$ . This diversification is achieved by the tax rate  $t_1^K$  onto the excess return, and the level of this tax rate depends – analogous to the reasoning given for the level of the tax rate  $t_2^L$  – on the strength of risk aversion in private consumption, relative to the one in public consumption.

However, there are two differences between diversifying wage risk and capital risk. First, optimal capital risk sharing can be implemented without any distortions in households' behavior and in private consumption. The reason for it is that it is possible to tax the risk premium directly, which causes only a Musgrave-substitution effect and leaves utility in private consumption unaffected (i.e., equation (20)). Second, households are already entirely diversified in capital risk, therefore, the government cannot improve private risk allocation. However, it can again provide an increased diversification of risk, because the provision of the public good, which is not provided by the capital market, increases the number of *socially* available assets. Again, the diversification result does not imply any assumption concerning the social discount rate. As the tax revenue is not redistributed as income, Gordon's (1985) neutrality result does not apply as well.<sup>19</sup>

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<sup>19</sup>See also the intuition given for Proposition 2 and section 2.



Next, we have to determine the optimal tax structure for ensuring the optimal risk diversification, derived in Propositions 2 and 3.

From  $\beta = t_0^K = 0$  in Proposition 1 and  $\pi_c(\tilde{W}^1) = \pi_G(\tilde{W}^1) = \pi(\tilde{W}^1)$  in Proposition 2, we can conclude the optimal wage tax structure and the optimal tuition fees. In the social optimum, we have

$$\beta^* = t_2^L \cdot (1 - \pi(\tilde{W}^1)) \cdot \bar{W}^1 \cdot g'(e^*) - (1 + r) \cdot (t_1^L \cdot W^0 - p_B) = 0, \quad (46)$$

and can add the first order condition (10) from the household's optimization problem in order to receive

$$(1 - \pi(\tilde{W}^1)) \cdot \bar{W}^1 \cdot g'(e^*) = (1 + r) \cdot W^0. \quad (47)$$

The LHS of equation (47) gives (social) marginal revenue of optimal educational investment, whereas the RHS shows its (social) marginal costs, which are equal to the wage forgone by attending university and bringing forward these costs into the second period of life.

Note that there is no tax term directly distorting marginal revenue and marginal costs, and that in case of aggregate risk society is not risk neutral, because risk cannot be eliminated by pooling. As the term  $\pi(\tilde{W}^1)$  mirrors optimal wage risk diversification between private and public consumption, equation (47) can be seen as stating production efficiency under uncertainty, because optimal human capital production is not distorted.<sup>20</sup>

Substituting (47) into (46) leaves us with

$$t_2^L - t_1^L = -\frac{p_B}{W^0}. \quad (48)$$

**Proposition 4.** *The differentiated wage tax and tuition fees are used in order to guarantee optimal risk diversification without distorting educational investment. Optimal wage taxation implies either progressive wage taxation  $t_2^L > t_1^L$  and education subsidies  $p_B < 0$  or regressive wage taxation  $t_2^L < t_1^L$  and tuition fees  $p_B > 0$ .*

*Proof.* The proof of Proposition 4 is directly taken from (48) and the fact that any

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<sup>20</sup>Production efficiency to be desirable in second-best models dates back to the analysis of Diamond and Mirrlees (1971). However, they restrict to the case of certainty.

distortive wage taxation is fully compensated by either tuition fees,  $p_B > 0$ , or education subsidies,  $p_B < 0$ . Remind that production efficiency thereby implies  $\frac{(1-\pi(\bar{W}^1)) \cdot \bar{W}^1 \cdot g'(e^*)}{1+r} = W^0$  from (47).

□

As the diversification depends on the strength of risk aversion in private consumption relative to that in public consumption, the tax rate  $t_2^L$  depends on this relative strength: The higher risk aversion in private consumption relative to that in public consumption, the higher the tax rate on the skill premium. The intuition is as follows: The more disutility in private consumption is caused by risk, relative to disutility in public consumption, the more wage risk should be transferred to public consumption.

If the risk aversion in private (public) consumption is sufficiently high (low), only progressive taxation  $t_2^L > t_1^L$  ensures optimal risk diversification. However, this must be complemented by an education subsidy  $p_B < 0$  in order to avoid disincentive effects on human capital investment, because progressive wage taxation implies *ceteris paribus* a tax burden on education. Thereby, the tax differential in percent should equal the ratio between the subsidy per semester,  $p_B$  and wage earnings per unit of time,  $W^0$ . If the household is little risk averse in private consumption, or the optimal tax rate  $t_1^L$  is very high because of the need to finance large public spending, the optimal wage tax structure can turn out to be regressive. In this case tuition fees  $p_B > 0$  are required to secure efficiency in allocation, as tax regression acts as education subsidy.

Proposition 4 fits to the results of optimal wage taxation in case of idiosyncratic risky human capital formation. If the risk is idiosyncratically distributed, the society itself is risk neutral in public consumption. Thus, the optimal surtax rate would be equal to one, if skilled labor was inelastic, and all disincentive effects could be controlled by education subsidies (see Schindler and Yang, 2007).

In a nutshell, a strong linkage between wage taxation and education policy is once more needed in order to improve or even restore efficiency, while the differentiated wage tax allows to follow another aim. This principle is well-known as ‘Siamese Twins’- concept by Bovenberg and Jacobs (2005).<sup>21</sup>

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<sup>21</sup>In Bovenberg and Jacobs (2005) this second aim is income redistribution, whilst in our paper

The optimal capital tax structure is implied by Propositions 1 and 3: The (virtually) riskless return  $r$  to each asset is tax-exempted,  $t_0^K = 0$ , and a positive tax rate  $t_1^K > 0$  is applied on the excess return  $\tilde{x} - r$  in the risky asset.<sup>22</sup>

Both the surtax rate on the skill premium,  $t_2^L$ , and the tax rate on the excess return,  $t_1^K$ , are determined solely by risk considerations. Looking at Propositions 2 and 3, we can infer that  $t_2^L, t_1^K \rightarrow 0$  if households are close to risk neutrality in private consumption and  $t_2^L = t_1^K = 1$  if households are risk neutral in public consumption.

Unfortunately, on this level of generality, it is very difficult to derive more clear-cut results or to provide explicit optimal taxation rules for  $t_2^L$  and  $t_1^K$ . Therefore, we will now assume that the technological shock  $\theta$  and with it the risky rate of return  $\tilde{x}$  and the skilled wage  $\tilde{W}^1$  are normally distributed. If so, private and public consumption,  $c_t$  and  $G_t$ , are normal, too, and we can apply a Rubinstein-Theorem in order to relate the optimal tax rates to global risk aversion  $GARA(a) = -\frac{E[U_{aa}]}{E[U_a]}$ ,  $a = c, G$ , in private and public consumption, as defined, e.g., in Varian (1992, p. 380).

From rearranging Propositions 2 and 3, it turns out

$$\frac{t_2^L}{1 - t_2^L} = \frac{GARA(c_t)}{GARA(G)} = \frac{t_1^K}{1 - t_1^K}, \quad (49)$$

and we conclude

**Proposition 5.** *The optimal tax rates for insurance (i) equally depend on the ratio of global risk aversion in private and public consumption, (ii) are increasing in risk aversion in private consumption and (iii) decrease in risk aversion in public consumption. This holds true at least as long the risky return to capital  $\tilde{x}$  and the skilled wage  $\tilde{W}^1$  are normally distributed.*

*Proof.* See Appendix 8.2 □

Equation (49) supports all the intuition and results concerning the tax rates  $t_2^L$  and  $t_1^K$ , given above for Propositions 2 to 4. Moreover, it provides an explicit tax

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the differentiation is required in order to secure an optimal diversification of risk between all social assets.

<sup>22</sup>It is straightforward to show that optimally  $t_1^K \in (0, 1)$ , because Proposition 3 and equation (45) cannot be fulfilled else.

rule, which can be seen as approximation for more general probability distributions of wages and capital returns.

Combining our results in Propositions 2 and 3 for general distributions again, optimal risk-sharing in wage and capital income finally implies

$$\frac{\pi_c(\tilde{W}^1)}{\pi_c(\tilde{x})} = \frac{\pi_G(\tilde{W}^1)}{\pi_G(\tilde{x})}, \quad (50)$$

and we end up with equal proportions of risk premia in human versus real capital. Accordingly:

**Corollary 1.** *There is an indirect risk diversification effect in wage and capital risk, equating the relative normalized risk premia  $\frac{\pi_a(\tilde{W}^1)}{\pi_a(\tilde{x})}$ ,  $a = c_t, G$  of human versus real capital investment between private and public consumption.*

Thus, the two relative ‘prices’ of human versus real capital risk, measured on the one hand by private consumption and on the other hand by public consumption, are balanced by an optimal tax and education policy.

Putting it all together, as long as leisure demand is inelastic, a tax system, incorporating a differentiated wage tax combined with either tuition fees or education subsidies and a risk tax on excess returns in real capital assets, can ensure efficient risk diversification of aggregate income risk between private and public consumption, whereby all risk premia are equated. Moreover, such a tax system does not cause any inefficiencies or distortions.

Hence, although human capital and real capital investment under risk are strongly linked, as shown by Williams (1978), and although neither the risk premium in wage income can be taxed separately, nor the households’ risk can be fully diversified (due to incomplete insurance markets regarding human capital risk), the presence of multiple aggregate risk does not cause major problems for an optimal tax and public policy – as long as the government has access to sufficient and suitable instruments.

However, this might change, if one introduces endogenous labor supply. In this case it is of importance that only the entire skill premium in human capital, but not the compensation for wage risk alone can be taxed.

## 6 Extensions and Omissions

To assume that leisure demand is exogenously given, is of course a very restrictive assumption. It is helpful to derive explicit solutions, because this is hardly possible, if there are multiple income risks and endogenous leisure demand, but we must be aware that we neglect substantial welfare costs of taxation. Therefore, we are going to provide some intuition on which effects should be expected in an extended setting, and we will thereby refer to results in other papers, using simplified models.

In case of endogenous leisure choice, it is well-known from Atkinson and Sandmo (1980) that it is optimal to balance overall excess-burden on distortions in labor supply and savings – except for special cases, where there is weak separability in leisure. Moreover, Jacobs (2005) shows that endogenous educational investment increases the elasticity of labor supply and therefore efficiency costs of labor taxation. Thus, implementing this in Atkinson and Sandmo (1980) should decrease the tax burden on labor. Next, Jacobs and Bovenberg (2005) state that, even in the case of weak separability in leisure, a positive tax burden on the (riskless) return to real capital can be useful as indirect instrument in order to mitigate distortions in human capital taxation.

However, all these papers focus on deterministic incomes. The model, most similar to the present one, is in Schindler (2006). He examines the optimal tax structure, using a proportional labor tax and the same capital tax system as we do, but focuses on a model, where only capital returns are risky in aggregate. While using endogenous labor supply, he neglects human capital investment and labor supply in the second period of life.

Schindler (2006) shows that the risk tax on the excess return allows to separate the risk issue and that in this case a twofold trade-off is emerging. First, the equivalent optimal taxation rule for deterministic labor and capital income like in Atkinson and Sandmo (1980) applies. This will most likely lead to underprovision of the public good. The latter determines the second trade-off. Underprovision can be countered by increasing the tax rate onto the excess return above the rate, which equates the risk premia in private and public consumption. This will generate more tax revenue in expected value and mitigate expected underprovision.

However, this has to be paid with increased risk in public consumption. Thus, the second trade-off is optimal risk diversification versus underprovision; there are no additional effects on private consumption.

Embedding all these results above into the results of this paper, allows to state a conjecture, how endogenous leisure demand will influence our conclusions:

**Conjecture 1.** *If leisure demand is endogenous in both periods of life, it is most likely that, compared to the results in section 5,*

- (i) *deterministic interest income will be taxed at a positive rate.*
- (ii) *the tax rate  $t_2^L$  on the skill premium will be decreased, and there is only suboptimal risk shifting to public consumption.*
- (iii) *progression of the wage tax will be increased (decreased), if unskilled labor supply is more (less) elastic than skilled labor supply. The opposite holds true in case of regressive taxation. Moreover, in case of progressive labor taxation, capital taxation acts as indirect subsidy to education, and education subsidies  $p_B$  should be expected to decrease. If labor taxation is regressive, instead, positive capital taxation should increase tuition fees.*
- (iv) *the tax rate  $t_1^K$  on the excess return should be increased, in order to generate more expected tax revenue and to mitigate the underprovision with the public good. This will be repaid by increased risk in public consumption, thus there will be too much social risk than compared to Proposition 3.*

The intuition for that conjecture is as follows: If distortions in labor supply cannot be avoided, it is optimal to balance the distortions over labor supply, savings and human capital investment. As taxation gets now more expensive, this will shift the trade-off away from risk diversification and towards efficiency. The major problem here is that there is no equivalent in wage taxation to the risk tax in real capital, which only targets the risk premium. Any wage tax will not only shift risk, but also cause disincentives, which cannot be fully controlled by educational policy.

Moreover, the more elastic a tax base is, the less should be its tax burden, then. This explains the first set of effects in part (iii). As capital taxation subsidizes

human capital investment, direct subsidies can decrease even more, as they would anyway. However, if the wage tax is regressive, subsidizing itself education, and tuition fees are used, the latter should be increased, in case there is positive capital taxation. The result in (iv) follows directly from the shift in the trade-off between efficiency and risk diversification in capital risk and the discussion of Schindler (2006).

Whereas introducing endogenous leisure demand seems to have strong effects on the results, it is straightforward to introduce several risky assets. This can be done by assuming several sectors employing both a risky technology and skilled labor. As long as the Markowitz-case can be applied, each household will then hold a fully diversified, identical market portfolio of risky assets. Taxing the excess return in each risky asset with the risk tax rate  $\tau_1^K$  will have the same effects as in the present model, where the risky asset can be interpreted as the market portfolio of all risky assets (see also Schindler, 2006, relying on Sandmo, 1977). In a nutshell, several risky assets should not change optimal public policy.

Another neglected item is unemployment risk. In fact, households are faced either with substantial unemployment risk or with risky income as unskilled worker. Due to competitive labor markets, our model cannot give any information about unemployment and education as insurance device. Of course, it is possible to model the flip side of the coin, stochastic unskilled labor income, but in our setting this is also of limited use, as households are unskilled in the first period only – before acquiring education. Although the absence of wage risk in the unskilled sector is on the one hand a deficiency of the model, it allows on the other hand for clear-cut results on optimal tax systems for skilled households.

## 7 Conclusions

We have shown that the government can provide efficient risk diversification between private and public consumption and that it can create an institution to ‘trade’ a part of uninsurable wage risk by using differentiated wage taxes and relying on adjusted educational policies. The simultaneous presence of risk in human and real capital does not challenge public policy very much, if it has access to a full set of instruments and if leisure demand is inelastic. This is in contrast to the

challenge in the private sector, where the effects of wage and capital risk differ substantially.

Our results fit into a growing literature, which emphasizes a strong linkage between optimal tax systems and educational policies. It turns out that in the presence of risk and sufficient risk aversion in private consumption, it is better to have ex-post tuition fees, thus, progressive wage taxation, which has to be accompanied by educational subsidies in order to stabilize human capital investment. This can be seen as a potential justification for most European education systems, traditionally not (very much) relying on tuition fees, but on progressive taxation – and sometimes even tending to offer public scholarships (i.e., in the Nordic countries).

## 8 Appendix

### 8.1 Proof of Proposition 1

*Proof.* We start by restating the household budget constraint as

$$\begin{aligned} p \cdot c_{-1}^* + \tilde{c}_t^* + p_e^{eff} \cdot e^* &= (1 - t_2^L) \cdot [\tilde{W}^1 \cdot g(e^*) - W^0] - t_1^L \cdot W^0 \\ &\quad + (1 - t_1^K)(\tilde{x} - r) \cdot A^{R*} + p \cdot (1 - t_1^L) \cdot W^0, \end{aligned} \quad (51)$$

where  $p_e^{eff} = p \cdot p_e = p \cdot [(1 - t_1^L) \cdot W^0 + p_B]$  is the effective (inflated) price of education and where the RHS of (51) mirrors total income, which has to be considered for the endowment effects, when the Slutsky decomposition is applied.



The required Slutsky decompositions are therefore

$$\frac{\partial c_{-1}}{\partial p_B} = \left[ S_{c_{-1}e} - e^* \cdot \frac{\partial c_{-1}}{\partial I} \right] \cdot p, \quad (52)$$

$$\frac{\partial c_{-1}}{\partial t_1^L} = (-W^0) \cdot S_{c_{-1}e} \cdot p - W^0 \cdot [1 + p \cdot (1 - e^*)] \cdot \frac{\partial c_{-1}}{\partial I}, \quad (53)$$

$$\begin{aligned} \frac{\partial c_{-1}}{\partial t_0^K} &= \frac{\partial c_{-1}}{\partial p} \cdot \frac{\partial p}{\partial t_0^K} + \frac{\partial c_{-1}}{\partial p_e^{eff}} \cdot \frac{\partial p_e^{eff}}{\partial t_0^K} \\ &= \left[ S_{c_{-1}c_{-1}} + p_e \cdot S_{c_{-1}e} + s^* \cdot \frac{\partial c_{-1}}{\partial I} \right] \cdot (-r), \end{aligned} \quad (54)$$

$$\frac{\partial e}{\partial p_B} = \left[ S_{ee} - e \cdot \frac{\partial e}{\partial I} \right] \cdot p, \quad (55)$$

$$\frac{\partial e}{\partial t_1^L} = (-W^0) \cdot S_{ee} \cdot p - W^0 \cdot [1 + p \cdot (1 - e^*)] \cdot \frac{\partial e}{\partial I}, \quad (56)$$

$$\begin{aligned} \frac{\partial e}{\partial t_0^K} &= \frac{\partial e}{\partial p} \cdot \frac{\partial p}{\partial t_0^K} + \frac{\partial e}{\partial p_e^{eff}} \cdot \frac{\partial p_e^{eff}}{\partial t_0^K} \\ &= \left[ S_{c_{-1}e} + p_e \cdot S_{ee} + s^* \cdot \frac{\partial c_{-1}}{\partial I} \right] \cdot (-r). \end{aligned} \quad (57)$$

Thereby,  $S_{ij}$  represents the substitution effect in demand for good  $i$ , if price  $j$  changes, and the partial derivative with respect to  $I$  indicates the corresponding income/endowment effect.

By replacing all derivatives in equations (38) and (39) by the expressions above, all income effects cancel out, and further simplification leaves us in (39) with

$$S_{c_{-1}e} \cdot \beta = S_{c_{-1}c_{-1}} \cdot t_0^K \cdot r. \quad (58)$$

Using (58) in order to simplify (38) even more, we end up with equation (59) as

$$S_{ee} \cdot \beta = S_{c_{-1}e} \cdot t_0^K \cdot r. \quad (59)$$

Combining (58) and (59) by substituting for  $\beta$ , results in

$$(S_{ee} \cdot S_{c_{-1}c_{-1}} - S_{c_{-1}e} \cdot S_{ec_{-1}}) \cdot t_0^K \cdot r = 0. \quad (60)$$

The first term in (60) is a principal minor of the substitution matrix, which is

known to be negative (semi-)definite. As long as we rule out semi-definiteness, this expression cannot be equal to zero, and consequently we infer  $t_0^K = 0$  from the second term in (60).

Inserting  $t_0^K = 0$  in the RHS of (59), we find  $\beta = 0$ , because the substitution effect in education with respect to the own price is negative,  $S_{ee} < 0$ .  $\square$

## 8.2 Proof to Proposition 5

*Proof.* If two stochastic variables  $\tilde{x}$  and  $\tilde{y}$  are bivariate normally distributed, a Rubinstein-theorem (see Rubinstein, 1976, p. 421f) can be applied:

$$\text{Cov}(z(\tilde{y}), \tilde{x}) = E[z'(\tilde{y})] \cdot \text{Cov}(\tilde{y}, \tilde{x}) \quad (61)$$

If the technological shock  $\theta$ , the asset return  $\tilde{x}$  and the skilled wage  $\tilde{W}^1$  are normal in our model, private consumption  $\tilde{c}_t$  and public consumption  $\tilde{G}_t$  are normally distributed as well (see Varian, 1992, p. 380). In this case, we can reformulate the covariances in equation (43) as

$$\text{Cov}(U_{c_t}, \tilde{W}^1) = E[U_{c_t c_t}] \cdot \text{Cov}(\tilde{c}_t, \tilde{W}^1), \quad (62)$$

$$\text{Cov}(V_G, \tilde{W}^1) = E[V_{GG}] \cdot \text{Cov}(\tilde{G}, \tilde{W}^1) \quad (63)$$

and in equation (45) as

$$\text{Cov}(U_{c_t}, \tilde{x}) = E[U_{c_t c_t}] \cdot \text{Cov}(\tilde{c}_t, \tilde{x}), \quad (64)$$

$$\text{Cov}(V_G, \tilde{x}) = E[V_{GG}] \cdot \text{Cov}(\tilde{G}, \tilde{x}). \quad (65)$$

Inserting the private budget constraint (6) for  $\tilde{c}_t$  respectively the public one (25) for  $\tilde{G}_t$  and applying some covariance rules, these covariances turn into

$$\text{Cov}(U_{c_t}, \tilde{W}^1) = E[U_{c_t c_t}] \{ (1 - t_2^L) g(e^*) \text{Cov}(\tilde{W}^1, \tilde{W}^1) + (1 - t_1^K) A^R \text{Cov}(\tilde{x}, \tilde{W}^1) \} \quad (66)$$

$$\text{Cov}(V_G, \tilde{W}^1) = E[V_{GG}] \{ t_2^L g(e^*) \text{Cov}(\tilde{W}^1, \tilde{W}^1) + t_1^K A^R \text{Cov}(\tilde{x}, \tilde{W}^1) \}, \quad (67)$$

$$\text{Cov}(U_{c_t}, \tilde{x}) = E[U_{c_t c_t}] \{ (1 - t_2^L) g(e^*) \text{Cov}(\tilde{W}^1, \tilde{x}) + (1 - t_1^K) A^R \text{Cov}(\tilde{x}, \tilde{x}) \}, \quad (68)$$

$$\text{Cov}(V_G, \tilde{x}) = E[V_{GG}] \{ t_2^L g(e^*) \text{Cov}(\tilde{W}^1, \tilde{x}) + t_1^K A^R \text{Cov}(\tilde{x}, \tilde{x}) \}. \quad (69)$$

From Proposition 3 and equation (45), we can equate the RHS of equations (68) and (69) and solve for  $t_1^K$  as

$$t_1^K = \frac{E[U_{c_t c_t}] A^R \text{Cov}(\tilde{x}, \tilde{x}) + \{E[U_{c_t c_t}] (1 - t_2^L) - E[V_{GG}] t_2^L\} g(e^*) \text{Cov}(\tilde{W}^1, \tilde{x})}{(E[U_{c_t c_t}] + E[V_G]) A^R \text{Cov}(\tilde{x}, \tilde{x})}. \quad (70)$$

Equating the RHS of equations (66) and (67), substituting (70) for  $t_1^K$  and collecting terms leads to

$$\begin{aligned} \frac{t_2^L}{1 - t_2^L} &= \frac{E[U_{c_t c_t}] \cdot \left\{ g(e^*) \text{Cov}(\tilde{W}^1, \tilde{W}^1) - \frac{(E[U_{c_t c_t}] + E[V_{GG}]) g(e^*) \text{Cov}(\tilde{W}^1, \tilde{x})^2}{(E[U_{c_t c_t}] + E[V_{GG}]) \text{Cov}(\tilde{x}, \tilde{x})} \right\}}{E[V_{GG}] \cdot \left\{ g(e^*) \text{Cov}(\tilde{W}^1, \tilde{W}^1) - \frac{(E[U_{c_t c_t}] + E[V_{GG}]) g(e^*) \text{Cov}(\tilde{W}^1, \tilde{x})^2}{(E[U_{c_t c_t}] + E[V_{GG}]) \text{Cov}(\tilde{x}, \tilde{x})} \right\}} \\ &= \frac{E[U_{c_t c_t}]}{E[V_{GG}]} = \frac{-\frac{E[U_{c_t c_t}]}{E[U_{c_t}]}}{-\frac{E[V_{GG}]}{E[V_G]}} = \frac{GARA(c_t)}{GARA(G)}, \end{aligned} \quad (71)$$

where, in the second line, we have used  $E[U_{c_t}] = E[V_G]$  from Proposition 2 and the definition of global absolute risk aversion  $GARA(a)$ ,  $a = c_t, G$ . This proves the first part of equation (49).

Analogously, we can apply Proposition 2 and equation (43) in order to solve the RHS of equations (66) and (67) for

$$t_2^L = \frac{E[U_{c_t c_t}] g(e^*) \text{Cov}(\tilde{W}^1, \tilde{W}^1) + \{E[U_{c_t c_t}] (1 - t_1^K) - E[V_{GG}] t_1^K\} A^R \text{Cov}(\tilde{W}^1, \tilde{x})}{(E[U_{c_t c_t}] + E[V_G]) g(e^*) \text{Cov}(\tilde{W}^1, \tilde{W}^1)}. \quad (72)$$

Equating the RHS of equations (68) and (69) and substituting (72) for  $t_2^L$ , now, we

have from the same procedure as above

$$\begin{aligned}
\frac{t_1^K}{1-t_1^K} &= \frac{E[U_{c_t c_t}] \cdot \left\{ A^R \text{Cov}(\tilde{x}, \tilde{x}) - \frac{(E[U_{c_t c_t}] + E[V_{GG}]) A^R \text{Cov}(\tilde{W}^1, \tilde{x})^2}{(E[U_{c_t c_t}] + E[V_{GG}]) \text{Cov}(\tilde{W}^1, \tilde{W}^1)} \right\}}{E[V_{GG}] \cdot \left\{ A^R \text{Cov}(\tilde{x}, \tilde{x}) - \frac{(E[U_{c_t c_t}] + E[V_{GG}]) A^R \text{Cov}(\tilde{W}^1, \tilde{x})^2}{(E[U_{c_t c_t}] + E[V_{GG}]) \text{Cov}(\tilde{W}^1, \tilde{W}^1)} \right\}} \\
&= \frac{E[U_{c_t c_t}]}{E[V_{GG}]} = \frac{-\frac{E[U_{c_t c_t}]}{E[U_{c_t}]}}{-\frac{E[V_{GG}]}{E[V_G]}} = \frac{GARA(c_t)}{GARA(G)}, \tag{73}
\end{aligned}$$

which proves the second part of equation (49).

Relying on equation (49) now, it is straightforward to prove Proposition 5, because its parts (ii) and (iii) follow from simple differentiation, taking  $GARA(c_t)$  and  $GARA(G)$  as parameters.  $\square$

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